Signal Processing Technique for Location Estimation

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Introduction

- Object location estimation is a very popular field, with many different methods of calculating an object's position.
- Problems arise when some sensor data acts as outliers.
- Current solutions to this problem either run into problems when there are too many outliers or are difficult to compute.
- The proposed solution uses algebra in conjunction with signal processing to find the object's location and by doing so the computation is simple while also providing accurate results.

Objectives

- Derive a solution to estimate an object's location using know sensor positions
- Compute and compare the results with other methods such as Maximum Likelihood (ML) and Two Step Least Squares Estimator (TSE).

Methods

- To estimate the object location, the minimum required number of measurements (d_i) needs to equal the number of unknowns.
- For the 2D case, the time it takes for the signal to travel from a transmitter to a receiver can be modeled with a simple expression related to the objects position

$$d_i = \|\boldsymbol{u}^0 - \boldsymbol{s}_i\| + \|\boldsymbol{u}^0 - \boldsymbol{s}_0\| + n_i \quad (1)$$

a) Where d_i is the elliptic curve for 2D space or ellipsoidal surface for 3D space. The amount of time for the signal to reach s_i.
b) $\boldsymbol{u}$ is the object position
c) s_i receiver
d) s_0 transmitter
e) n_i measured noise

$$2d_i\|\boldsymbol{u} - \boldsymbol{s_o}\| = d_i^2 + \|\boldsymbol{s_o}\|^2 - \|\boldsymbol{s_i}\|^2 \quad (2)$$

a) Further simplification gives $\|\boldsymbol{u} - \boldsymbol{s_o}\| = \boldsymbol{\alpha_i^T u} + k_i$ where $\boldsymbol{\alpha_i} = \frac{1}{d_i}(\boldsymbol{s_i} - \boldsymbol{s_o})$, $k_i = \frac{1}{2d_i}(d_i^2 + \|\boldsymbol{s_o}\|^2 - \|\boldsymbol{s_i}\|^2)$

$$[\boldsymbol{u}^t \quad 1]\boldsymbol{A}\begin{bmatrix}\boldsymbol{u}\\ 1\end{bmatrix} = 0,\ \boldsymbol{A} = \begin{bmatrix}\boldsymbol{\alpha_i\alpha_i^T - I} & k_i\boldsymbol{\alpha_i} + \boldsymbol{s_o}\\ k_i\boldsymbol{\alpha_i^T} + \boldsymbol{s_o^T} & k_i^2 - \|\boldsymbol{s_o}\|^2\end{bmatrix} \quad (3)$$

a) Equation 3 above now sets up a quadratic where the roots are the estimated positions.

$$[(\boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2)^T \quad k_1 - k_2]\,[\boldsymbol{u}^T \quad 1]^T = 0 \quad (4)$$

a) Equation of dotted line in Figure 1.

- By adding equation (3) and (4) we will be able to find the roots.

$$\frac{-h(1,2) \pm \sqrt{h(1,2)^2 - h(1,1)h(2,2)}}{h(1,1)},\ h(1,1) \neq 0$$

$$-\frac{h(2,2)}{2h(1,2)},\ h(1,1) = 0$$

- The object location estimate is the intersection point of the curves from the algebraic expressions and it involves only finding the roots of quadratic equations.
- A similar procedure can be taken for 3D localization

$$\boldsymbol{h} = \boldsymbol{H}\boldsymbol{u}^{\circ} + \boldsymbol{B}\boldsymbol{\epsilon} \quad (5)$$

- Equation (5) is derived from getting the individual solutions of $\mathbf{u}$

$$\boldsymbol{u} = (\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{W}\boldsymbol{h} \quad (6)$$

- Applying Gauss-Markov theorem, u can be calculated which is the solution to BLUE.

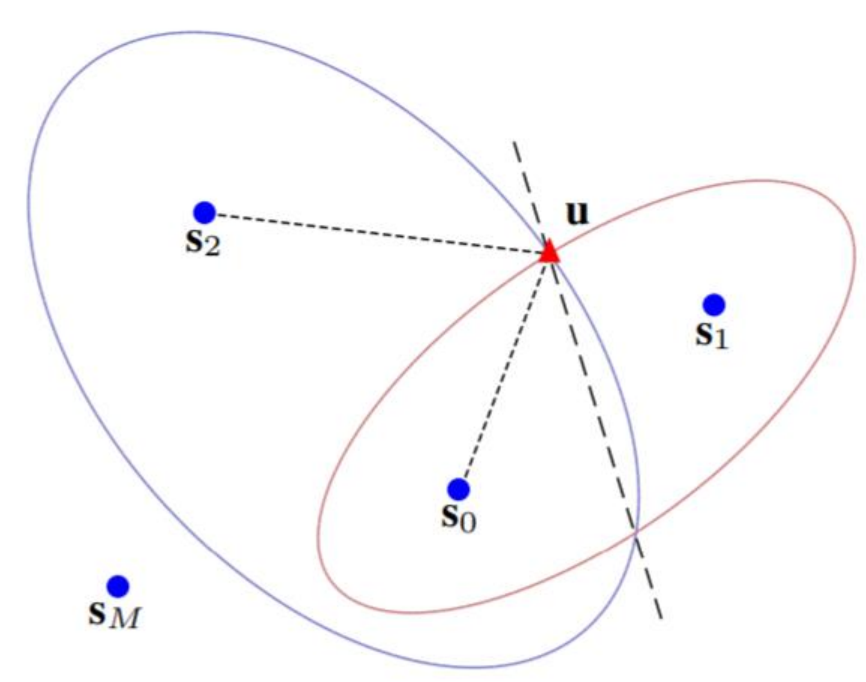

Figure 1: The intersection point **u** is the estimate of the object's location

Results

- MATLAB was used along with sample data to help estimate the object location

BLUE	0.1894	0.6238	1.9164	6.0572	19.0903	60.9237	586.0006
TSE	0.1895	0.6245	1.9187	6.0941	19.5072	77.1208	953.2679
IMLE	0.1894	0.6238	1.9163	6.0573	19.0930	60.9060	389.5717

Figure 2: Comparing distances between proposed solution Best Linear Unbiased Estimate (BLUE), Maximum Likelihood Estimate (ML) and Two Step Least Squares Estimate (CF)

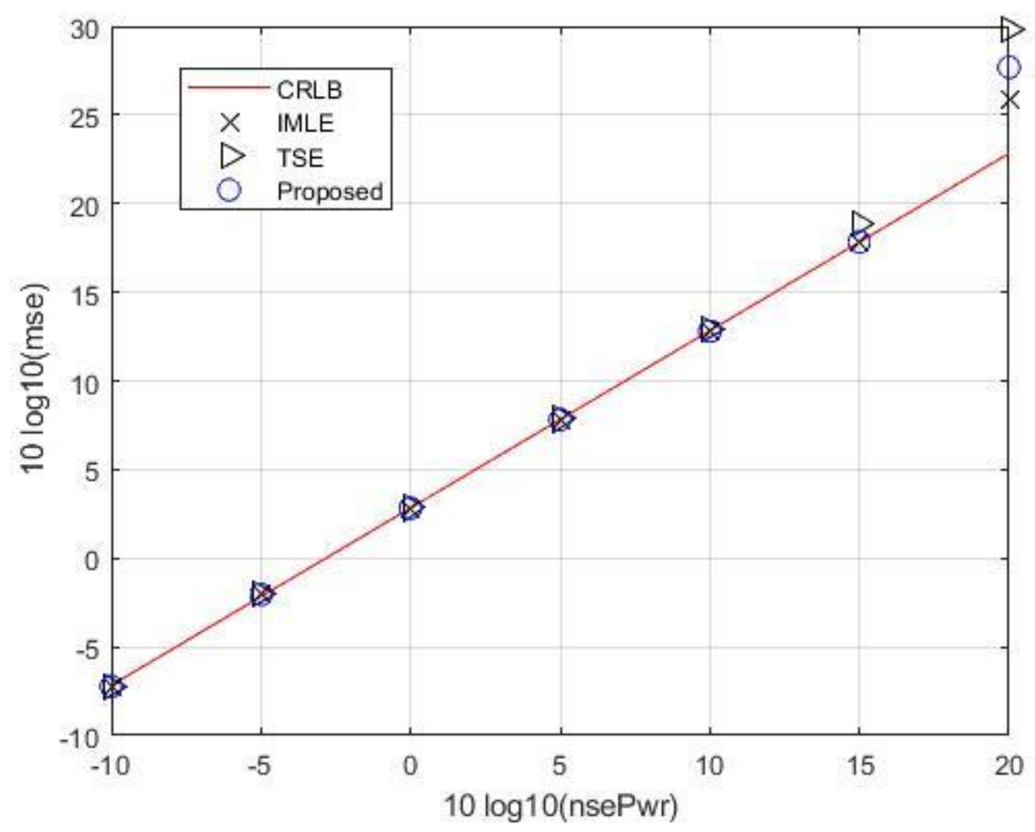

Figure 3: Graph of the estimated object location of different methods

- As can be seen in Figure 2, the BLUE method results in numbers very close to the other estimates
- When looking at the graph, the proposed solution, BLUE, lies almost exactly on the same points as the other estimates.

Conclusions

- The proposed method makes it easy to obtain accurate estimates of an object's location
- It is computationally easy requiring a combination of signal processing and algebra

References

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