Investigating the structure of Pyramid Tessellated Surfaces

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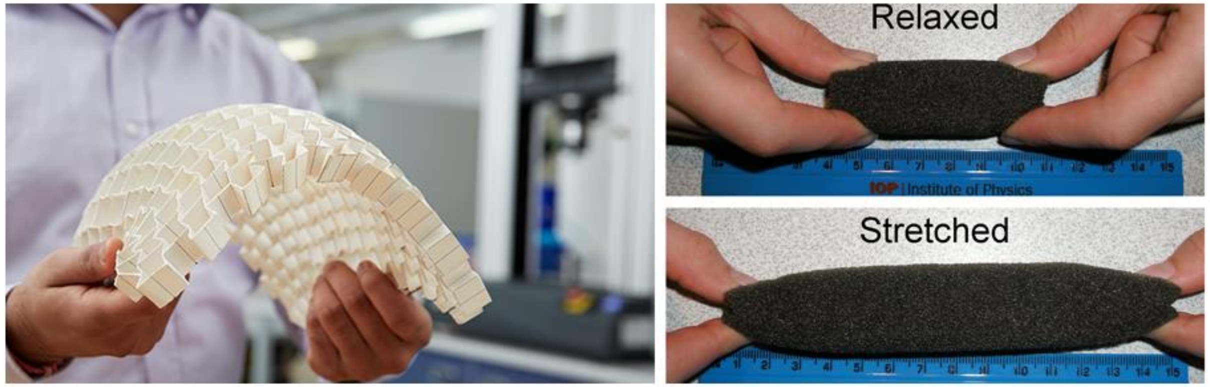
What are Metamaterials?

- Typically characterized by a certain geometry known as a unit cell that is arranged
in a repeating pattern or lattice.
- Manufactured as opposed to naturally occurring.
- Produces properties that naturally occurring materials do not have.

A photonic metamaterial that displays a negative refractive index.

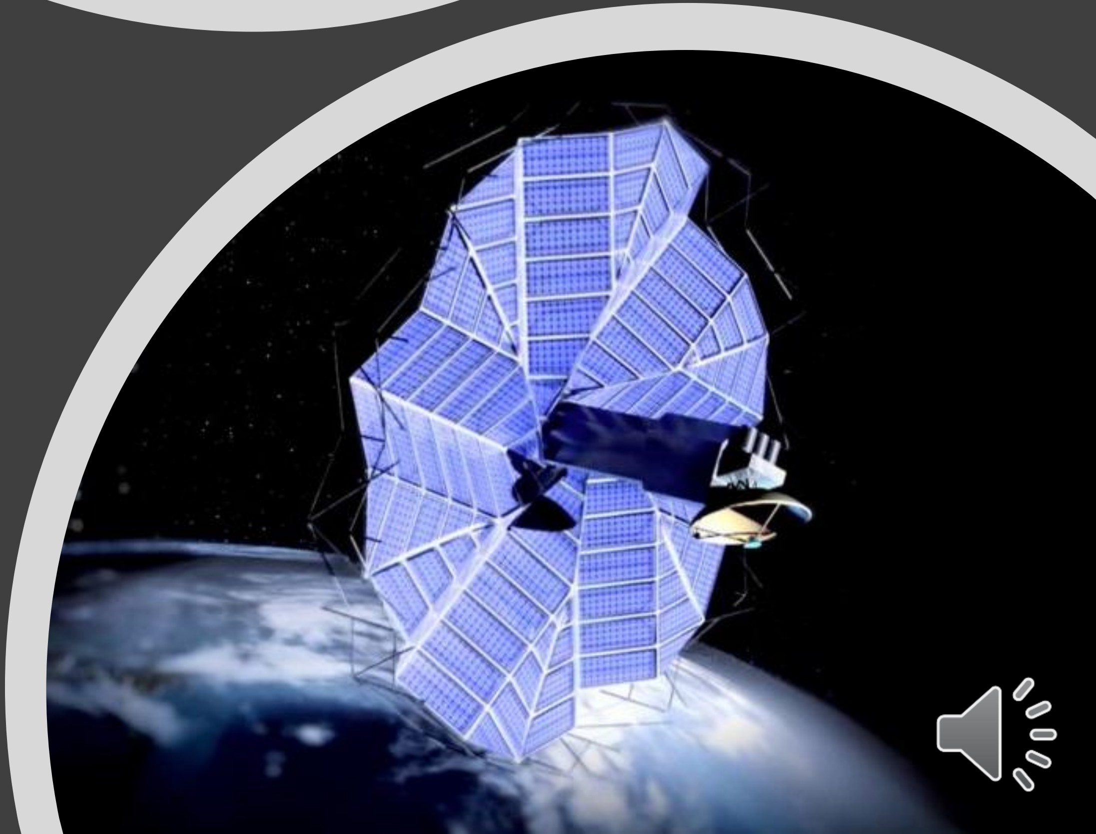


A mechanical metamaterial that displays a negative Poisson’s ratio.

A Miura surface is amongst the most popular in origami inspired structures.

Origami inspired structure…

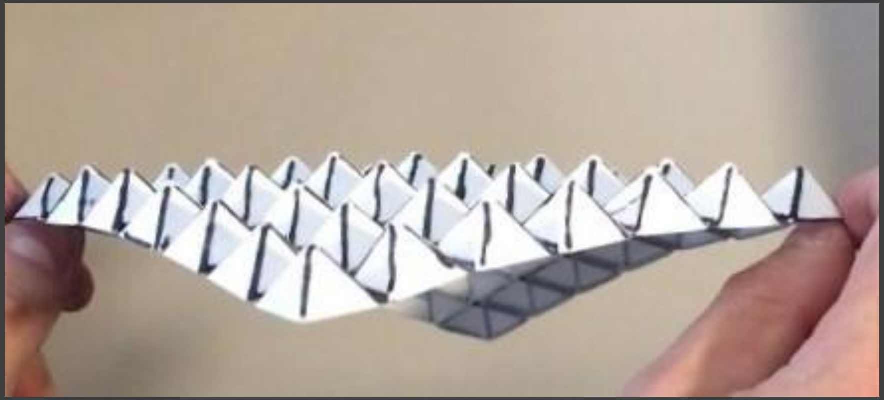
- Aim to utilize techniques of origami to approach structural and machine design.
- Folding properties are highly desired for purpose of deployable structures.
- The transformative and “morphing” property of certain origami structures can allow for structures to respond dynamically to their environment if so desired.

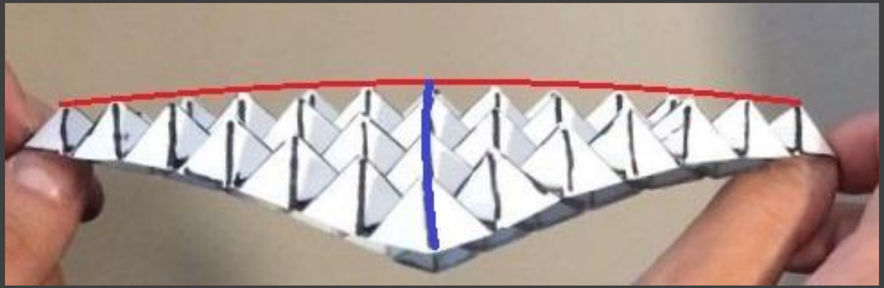


Deployable solar panels an example of the folding nature of origami structures being utilized.

Objectives

- Develop a model and MATLAB code that can correctly simulate the pyramid tessellated surface.
- Utilize simulations to predict behavior of the surface in response to different energy conditions.
- Explore potential applications which invoke the pyramid tessellated geometry.





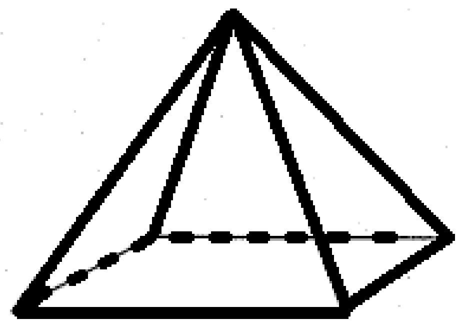
- In response to strain along the diagonal, two orthogonal bending moments are produced.
- A transition from a flat to a curved surface occurs, specifically, a change within the intrinsic or Gaussian curvature occurs in response to the strain.

The Strategy

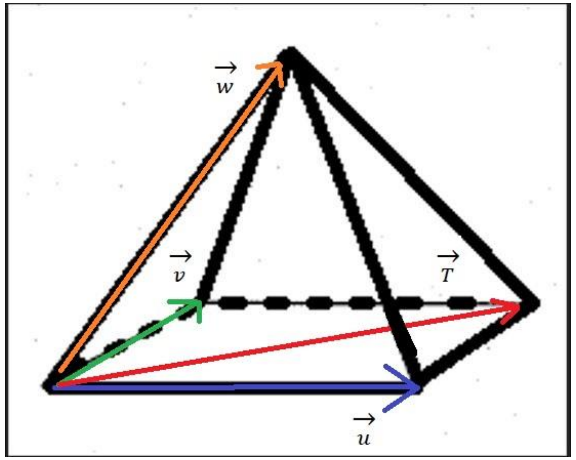
- Start by modeling a single unit cell.
- Build the entire surface through the use of boundary conditions and recursion.
- Define the sources of energy in the structure, and then use energy minimization approaches to find the boundary conditions that correspond to the minimum energy given certain conditions(i.e. stretching along the diagonal).

The Unit Cell

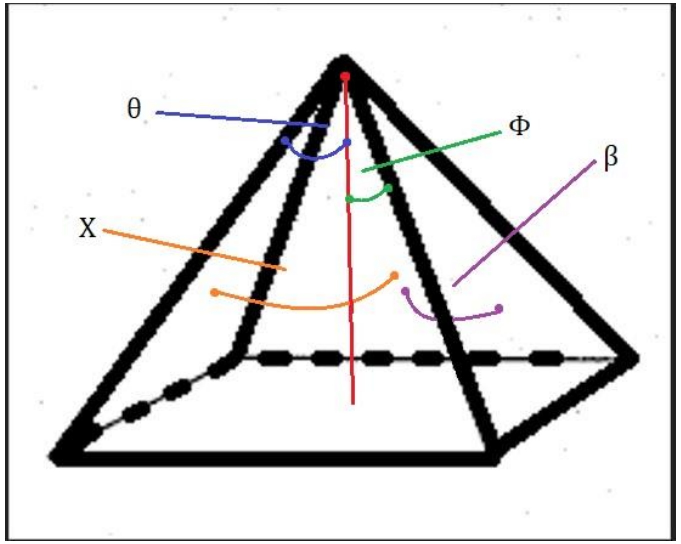
- Assumptions: The pyramid is an equilateral square pyramid(all side lengths are equal). Side lengths are unit length and rigid, experiencing no deformation.
- Energy within a unit cell is modeled as a torsional spring with respect to the dihedral angles χ and β.
- Basis vectors u and v along with the angle θ are enough to characterize the geometry of the entire cell, from which w and T can be constructed.



Individual Unit Cell



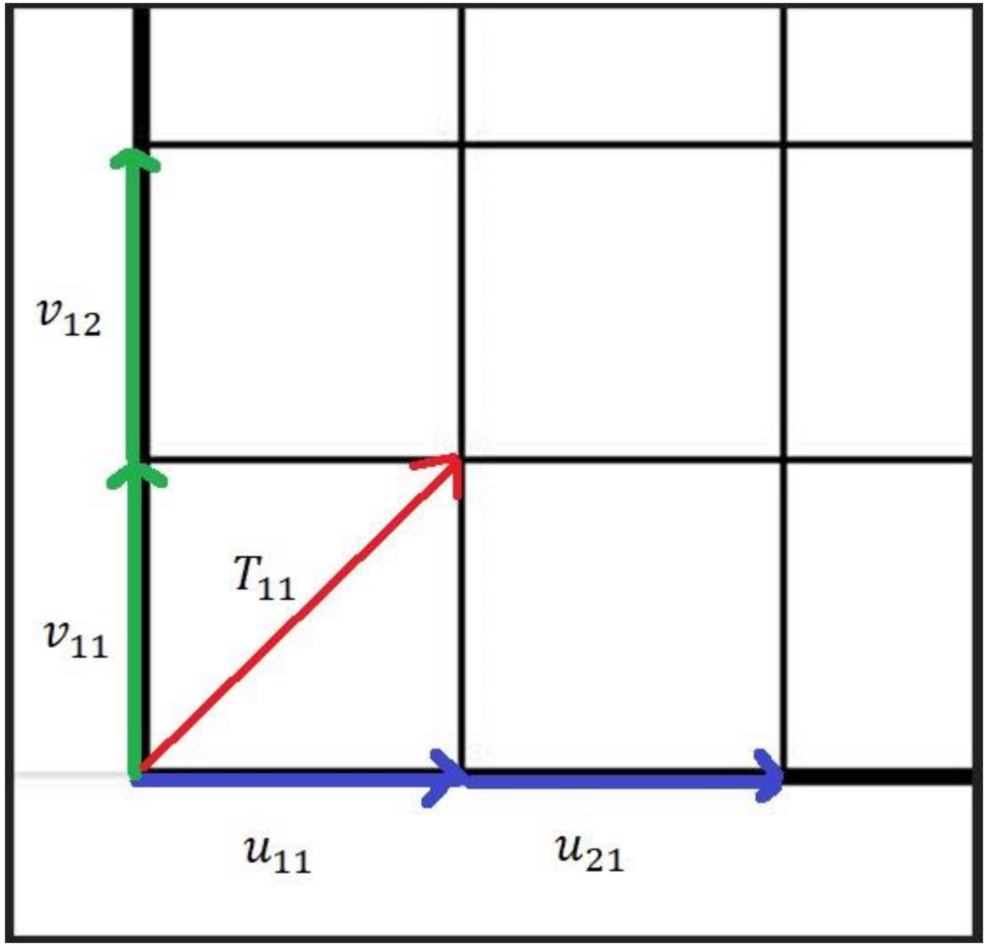
Unit Cell vectors.



Angles necessary for characterizing Energy and Position.

Constructing the sheet

- Given a set of Boundary Condition vectors that describe adjacent sides of the square sheet, we can construct the entirety of the sheet recursively.



First 2 by 2 section of a sheet illustrated to demonstrate recursive principle

B.C. 1 = u_{m1}

B.C. 2 = v_{1n}

$T_{m,n} = f(u_{m,n}, v_{m,n})$

General formula for recursive construction

$$u_{mn} = T_{m,n-1} - v_{m,n-1}$$

$$v_{mn} = T_{m-1,n} - u_{m-1,n}$$

Energy model/ minimization

- Given the rigid length assumption, energy is modeled as torsional springs through angles which define individual unit cells and the angles between unit cells.
- To correctly simulate the behavior of the sheet, we must minimize the energy with respect to a given condition. For the scenario of stretching along the diagonal of the sheet, we will choose a condition that describes elongation along that diagonal.
- Minimization is achieved through the approach of gradient descent.

General formula for gradient descent.

$$x_{i+1} = x_i - \alpha \nabla G$$

The form of the equation that we will be implementing.

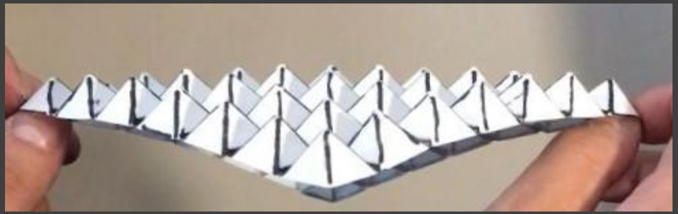
$$x_{i+1} = x_i - \alpha \nabla E \; - \; \gamma \nabla H$$

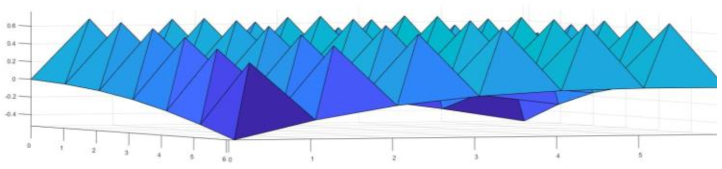
(The function E describes energy while H describes the stretched condition.)

Results

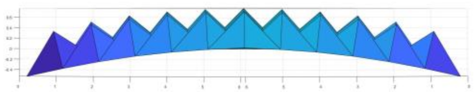
- Multiple iterations of trial and error slowly improved the gradient descent's efficiency at energy minimization.
- Characteristic two orthogonal bending moments produced.

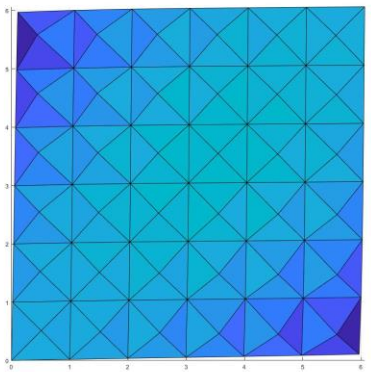
Previous photos showing stretched response

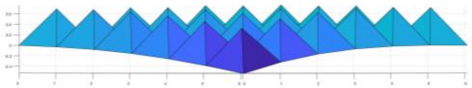




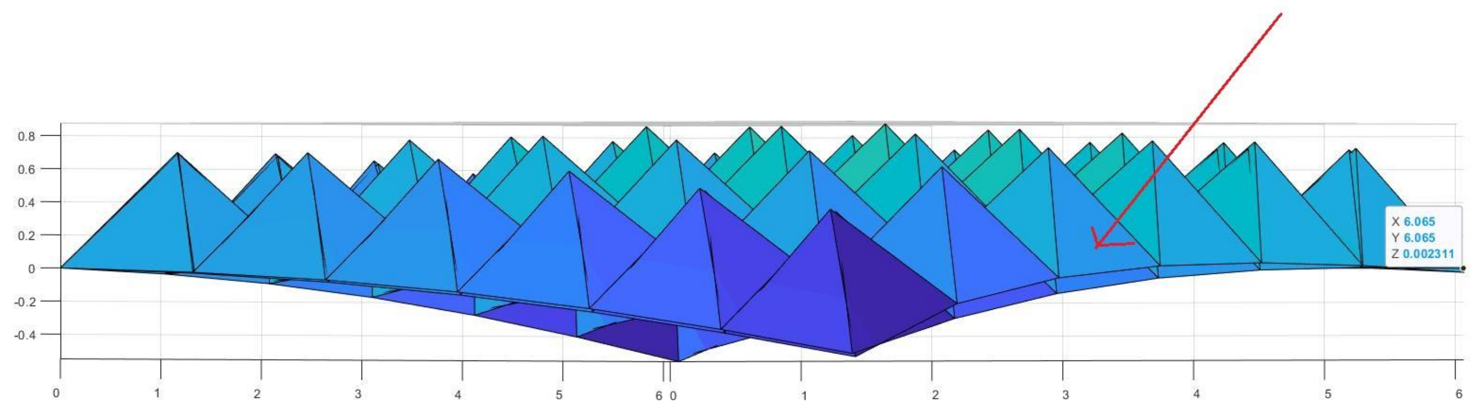
6 by 6 sheet with a 0.01 strain along the diagonal.



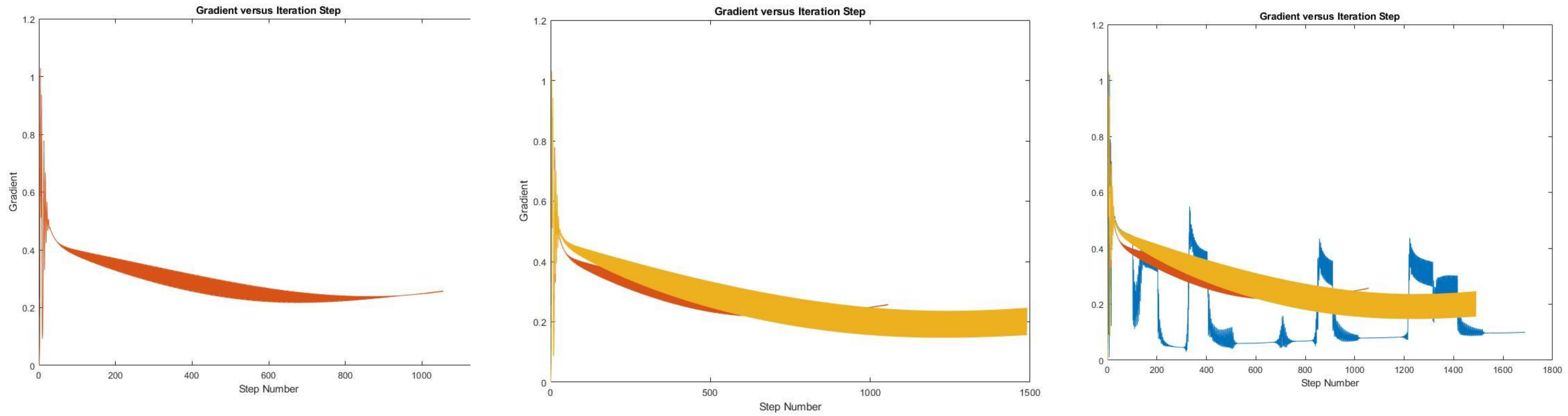




- The red arrow highlights the unminimized energy surface, where more asymmetry and higher curvature can be observed.



Two surface plots. One of the unminimized energy, the other of the best achieved minimum energy.



Red = Unminimized, **Yellow** = Normal Energy minimization technique, **Blue** = Energy minimization using Std. Dev. Method.

-Satisfying the stretching constraint in the problem naturally produced high frequency oscillations within the gradient.

-Oscillations were still present when the energy minimization constraint is introduced.

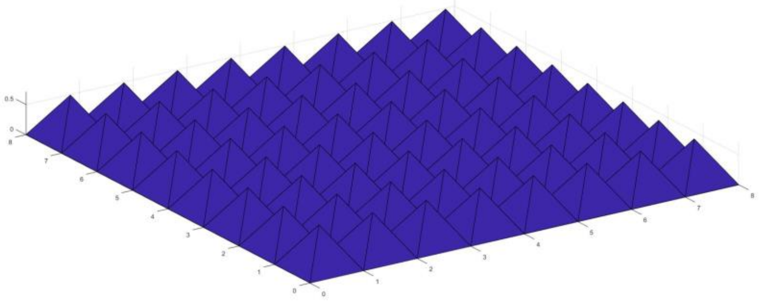
-The best approach to minimizing the energy with the given stretching utilized a sampling method that adapts the learning rate based on the standard deviation of the last n iterations.

Final Energy values of the sheet for each method

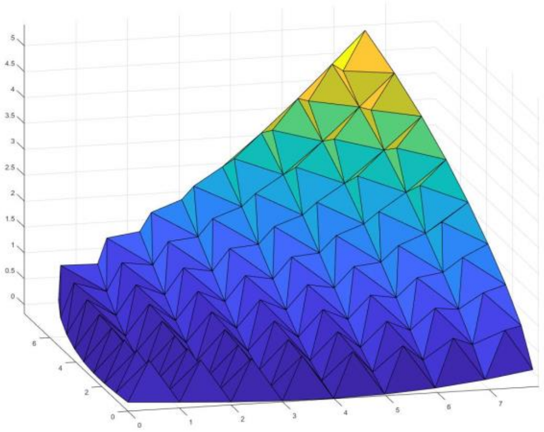
-Red = 0.337 (unminimized)

-Yellow = 0.2237 (33% improvement)

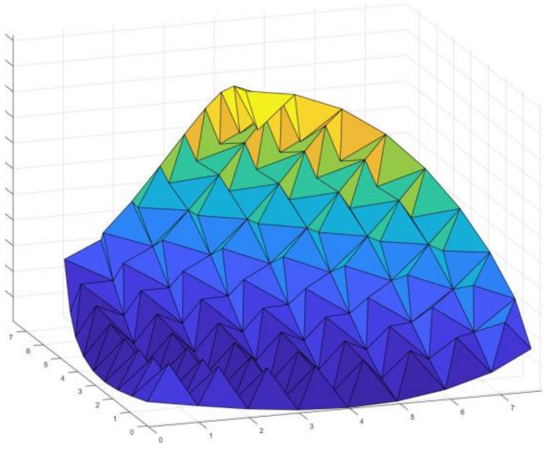
-Blue = 0.1808 (46.3% improvement)



χ = 0

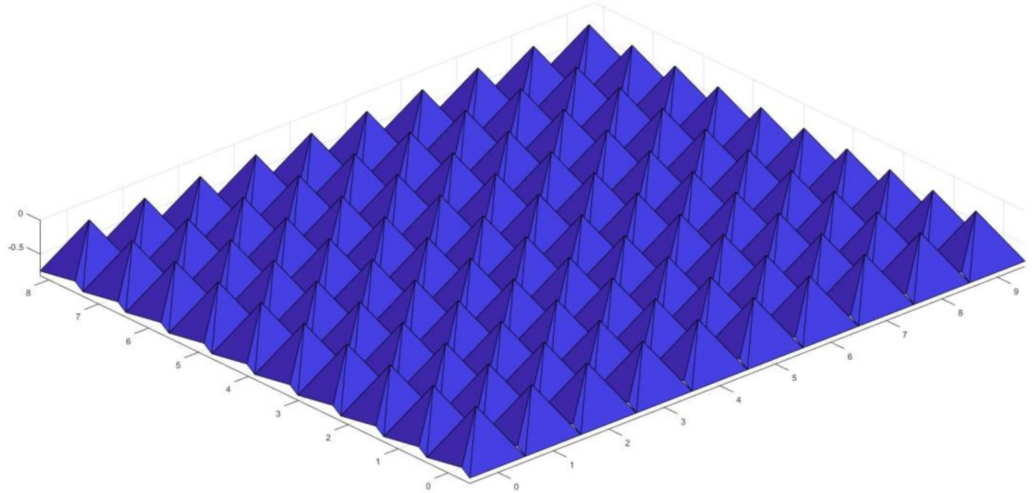


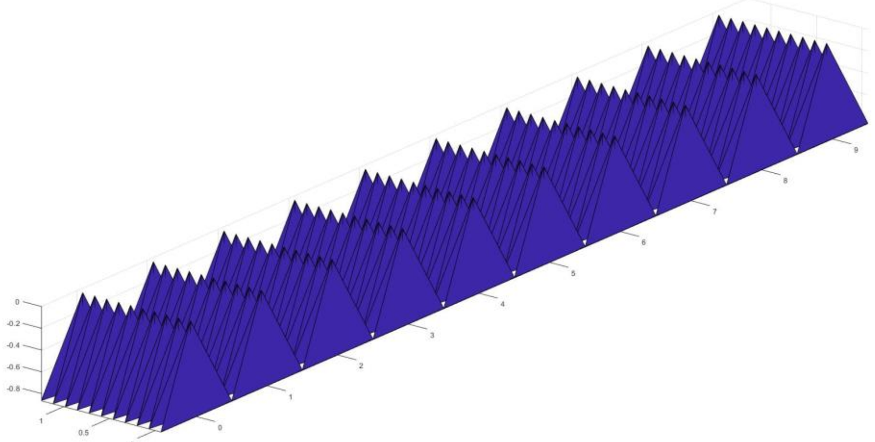
χ = 0.3



χ = 0.6

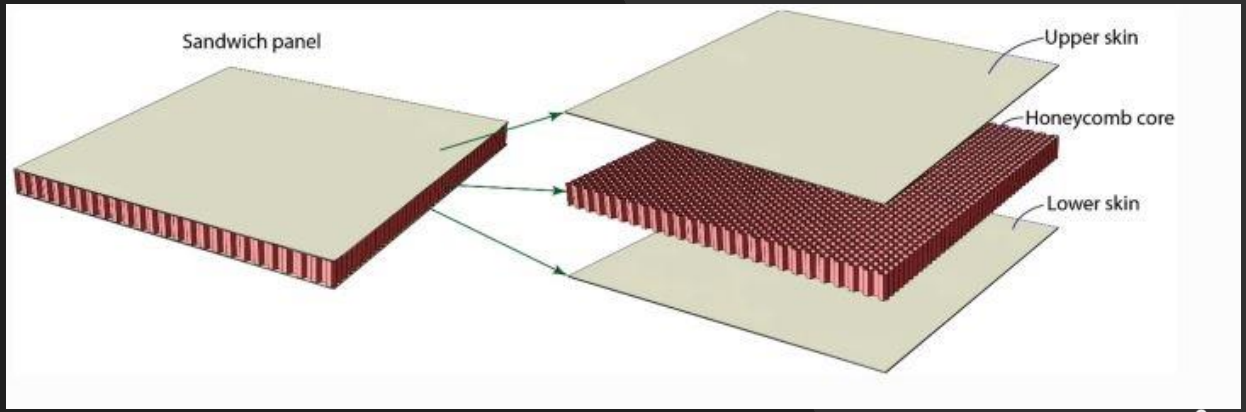
- Predicting possible states through energy minimization in an 8 by 8 sheet under different constraints.
- The parameter(χ) governing the constraint, that varies in each diagram, is the dihedral angle corresponding to the minimum energy state of each unit cell.
- χ = 0 represents a minimum energy state corresponding to the default square pyramid state. Higher values of χ correspond to a minimum energy state of a unit cell that is more closed.
- Increasing χ gradually produces negative Gaussian curvature.

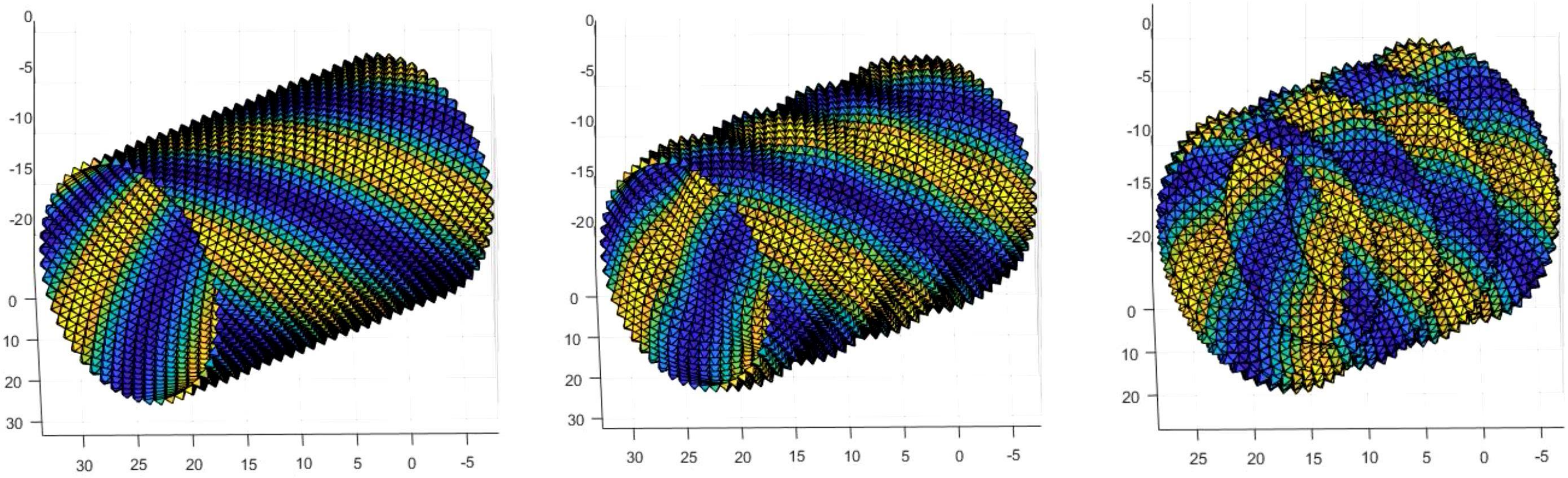




A Deployable Frame

- A folding frame can be developed if new degrees of freedom are added to each unit cell by allowing opposite faces of the pyramid to fold.
- Motion would still be restricted to folding where both sides of the unit cell must fold in the same way, this allows for sheet to only fold and unfold in one way.
- The frame would be durable relative to stresses that are applied perpendicular to its direction of folding. An example being stresses applied to the top of the frame, pointing downward. This would allow for deployable frames for structures that need to resist the same type of stresses, like sandwich panels.



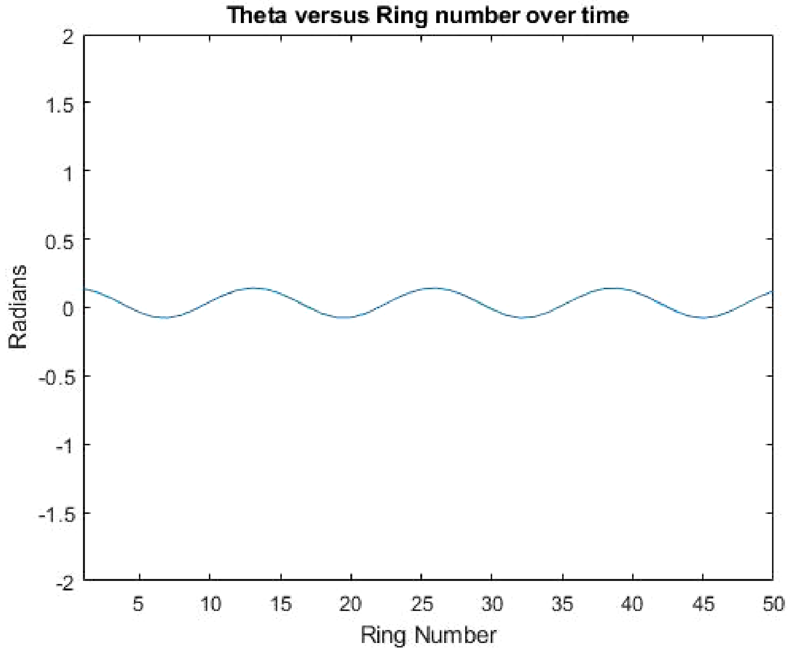


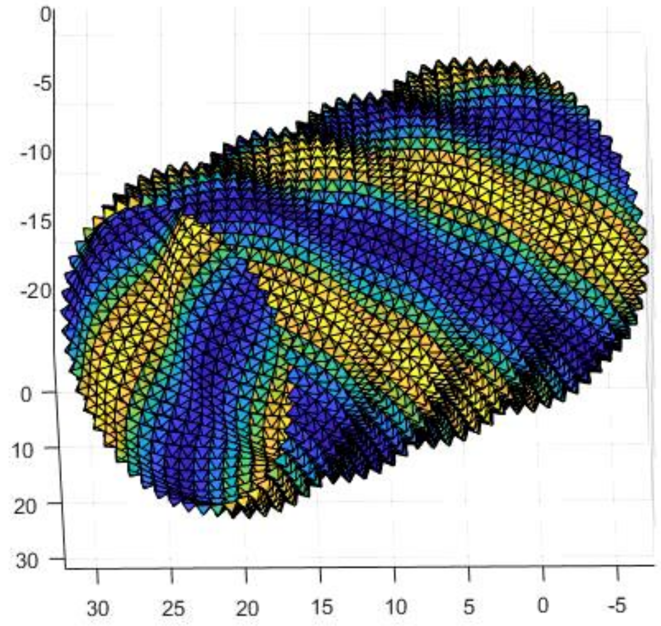
50 by 50 cylinder(50 unit cells around, 50 unit cells long)

Curious behavior when embedded…

- Wrapping the pyramid sheet around onto a tube in a particular way produces interesting dynamics.
- Varying the angles of the unit cells oriented along certain rings by the same amount results in contraction.

- The angles corresponding to the unit cells around a ring vary periodically down the cylinder when contraction occurs.
- Contraction of the unit cell angles in a certain ring reduces the circumference of that ring, while opening of the unit cell angles increases the circumference. These two behaviors help characterize the contraction motion seen axially along the cylinder.





Conclusion and the Next Step

- Energy minimization, even through gradient descent, is deceivingly difficult. Introducing very simple constraints can quickly make naïve gradient descent insufficient, and more sophisticated techniques such as sampling methods and stochastic methods are needed.
- The pyramid tessellated surface produces intriguing behaviors. While we have produced predictive results of the final states that the surface achieves under certain conditions, in general, the actual dynamics of this surface is not understood and has yet to be modeled.
- A more in depth analysis of the deployable frame is needed to determine whether or not it is structurally valid. A design for the skin is also needed…
- The pyramid tessellated cylinder, while modeled, still desires further understanding. It's ability to contract in response to the actuation of certain rings is unique. Ideas for applications are still under construction and need further investigation.
- A truly solid mathematical framework for how the pyramid tessellated surface behaves is not yet complete, however some results currently look promising. A major goal will be to establish this framework utilizing the language of differential geometry.

Acknowledgments…

University of Missouri

- I'd like to thank the University of Missouri and the College of Engineering for giving me this valuable research opportunity. I have developed a passion for the process and journey of research.
- My professor, Dr. Nassar, has been incredibly helpful to me as a mentor. He has always been available for help when I need it and has helped me grow as a student. I have enjoyed working with him and hope to continue doing so.