Nonlinear Dynamics: a look into one-dimensional and two-dimensional flows

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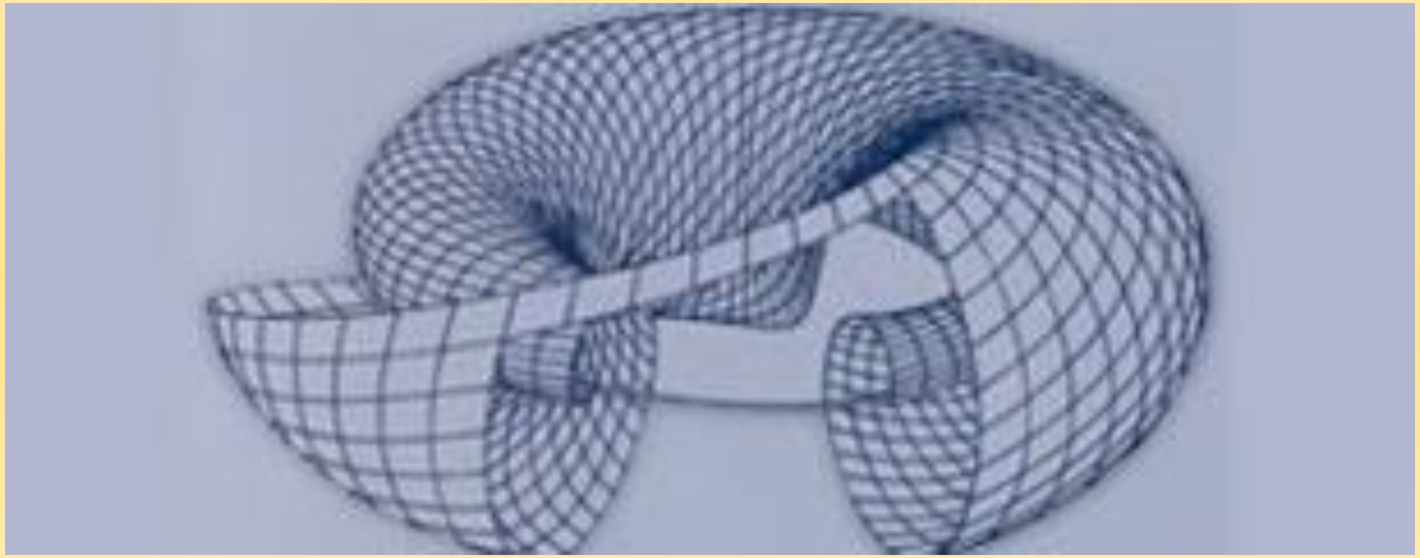
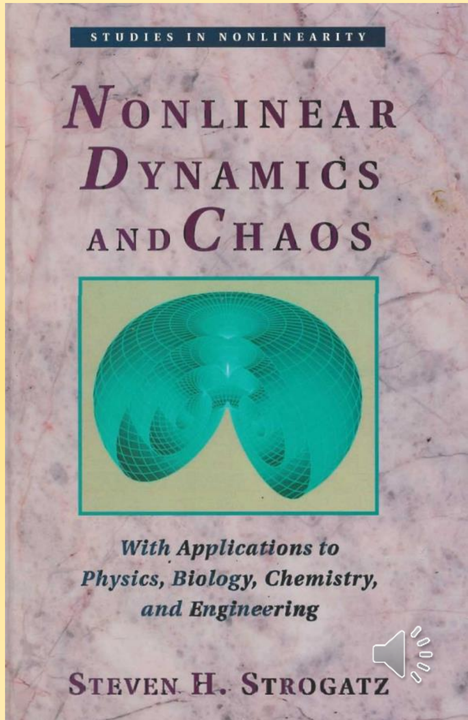




Background information

- Dynamics deals with systems that evolve over time
  - Nonlinear examples (products, powers, and functions of x)
- The goal is to study nonlinear dynamics from a bottom-up perspective
  - One-dimensional
  - Two-dimensional
  - Three-dimensional and higher (Chaos theory, fractals, etc.)

- All figures/diagrams taken from the book *Nonlinear Dynamics and Chaos*

Fixed points and points of stability

Fixed point

- Fixed points are found where $\dot{x} = 0$ (y = 0)

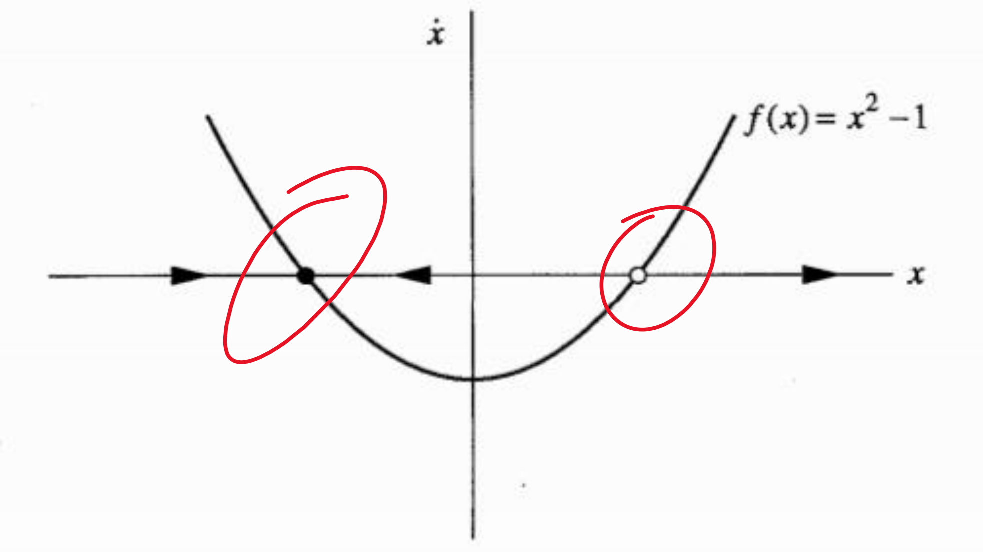


Figure 2.2.2

Points of Stability

Stable points

- A fixed point that is known as an attractor
  - Drawn in as a “filled-in point”

Unstable points

- A fixed point that’s flow is known as a repeller
  - Drawn in as an “empty” point

Population Model/Example

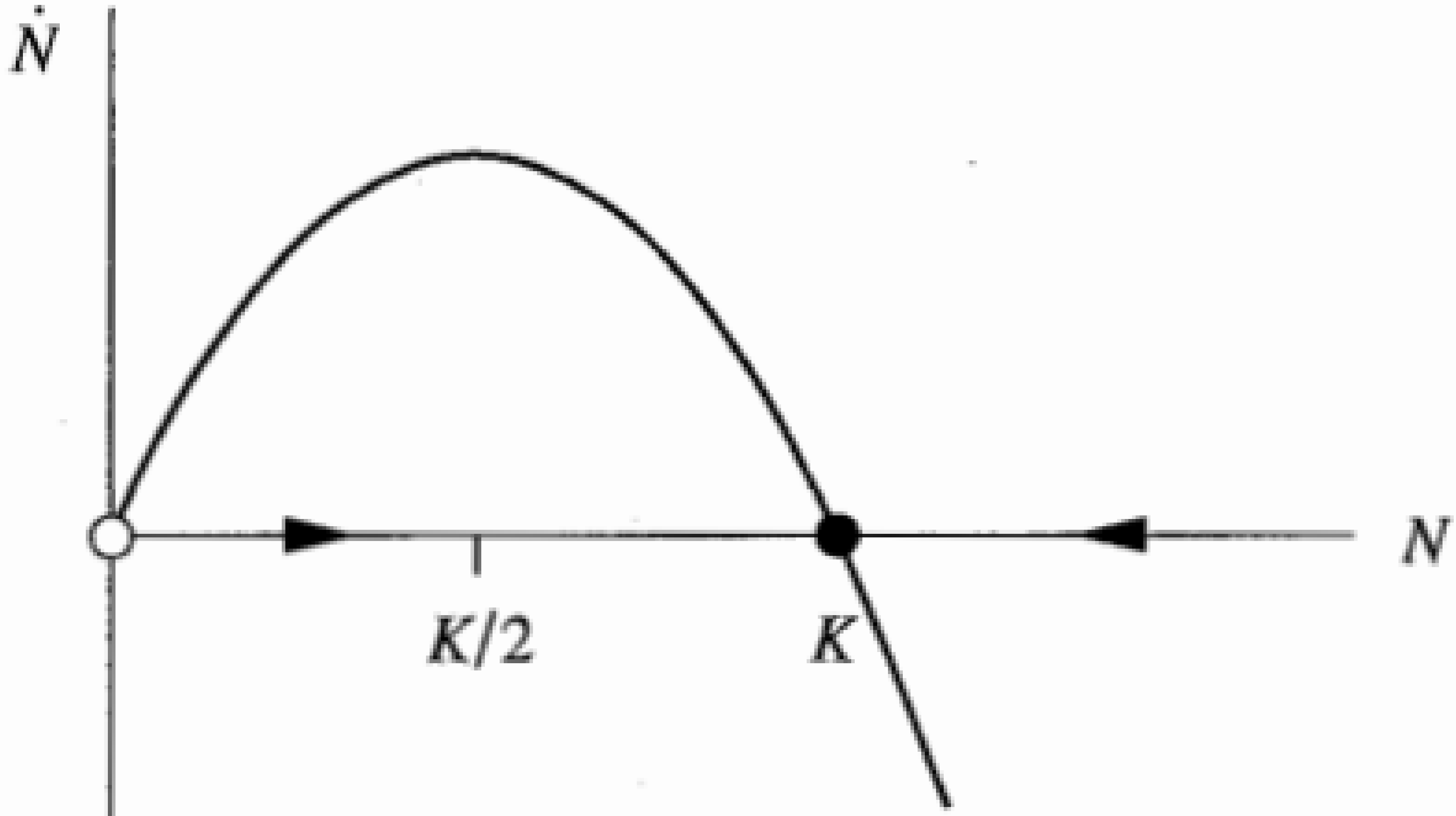


Figure 2.3.3

Bifurcations

- Bifurcations are qualitative changes in the dynamics of a model, problem/example, graph, etc.
  - I.e., fixed points can be created, destroyed, stability changes, etc.

- Types of Bifurcations:
  - Saddle-node
  - Transcritical
  - Supercritical Pitchfork

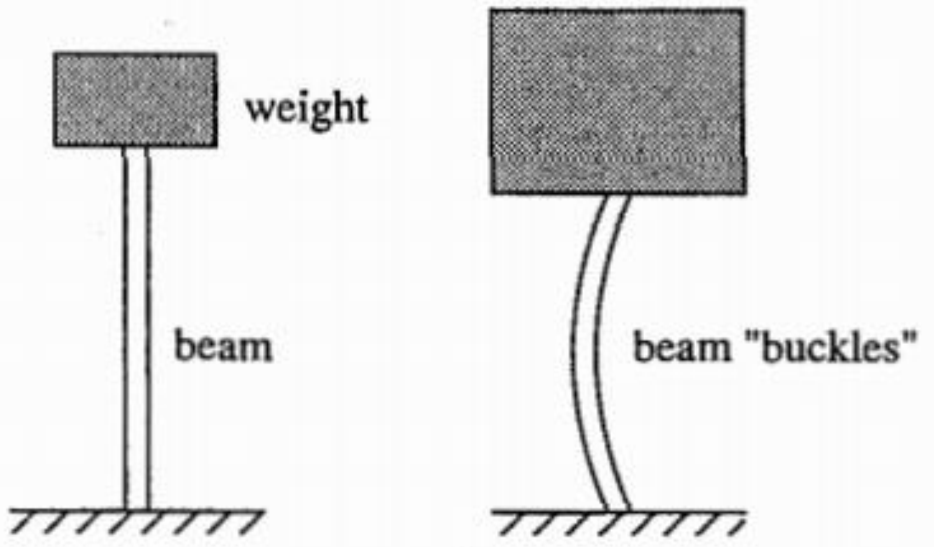


Figure 3.0.1

Types of Bifurcations with models

Saddle-node Bifurcation

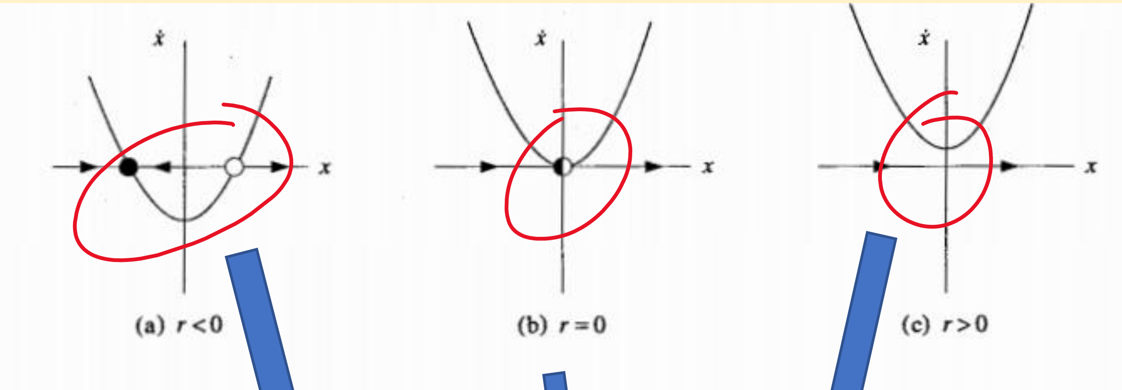


Figure 3.1.1

$\dot{\mathbf{x}} = r + x^2$

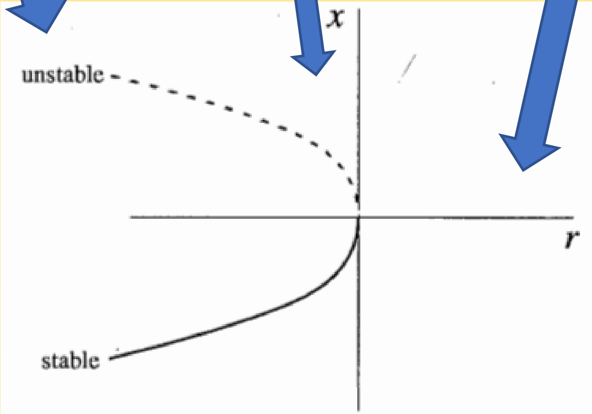


Figure 3.1.4

Transcritical

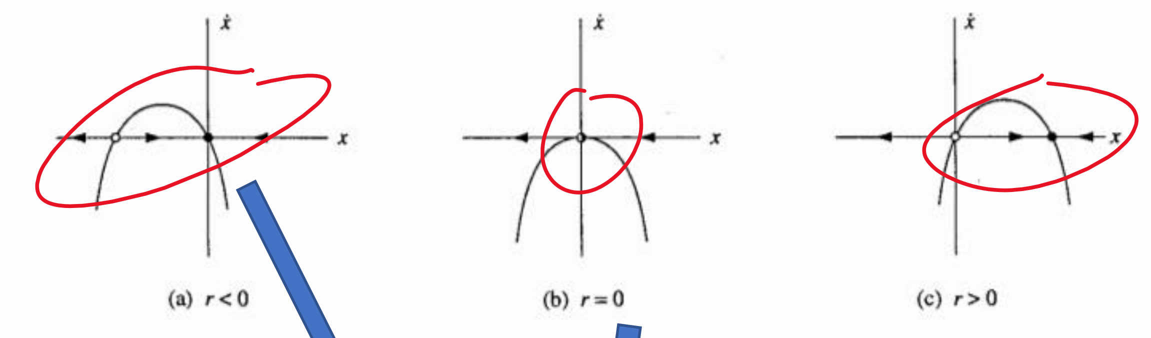


Figure 3.2.1

$\dot{\mathbf{x}} = rx - x^2$

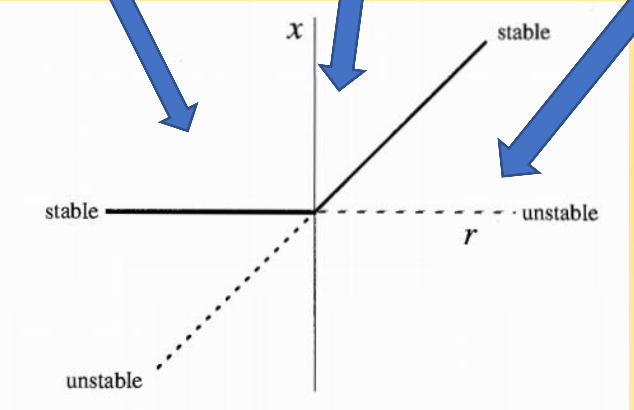


Figure 3.2.2

Types of Bifurcations with models

Supercritical Pitchfork Bifurcation

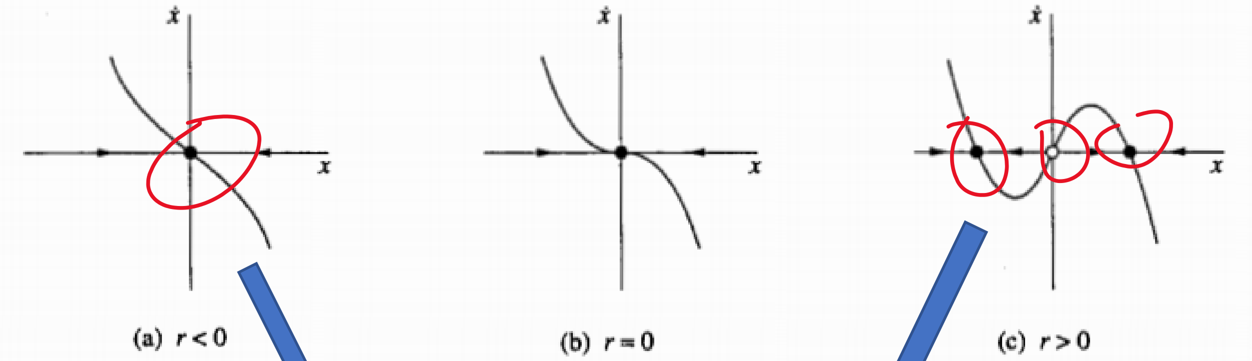


Figure 3.4.1

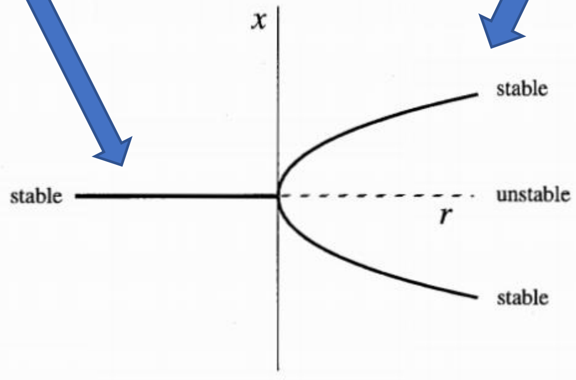


Figure 3.4.2

Introduction to phase planes and phase portraits (2-D)

- Phase planes are 2-D images of how a certain example will play out over time using trajectories

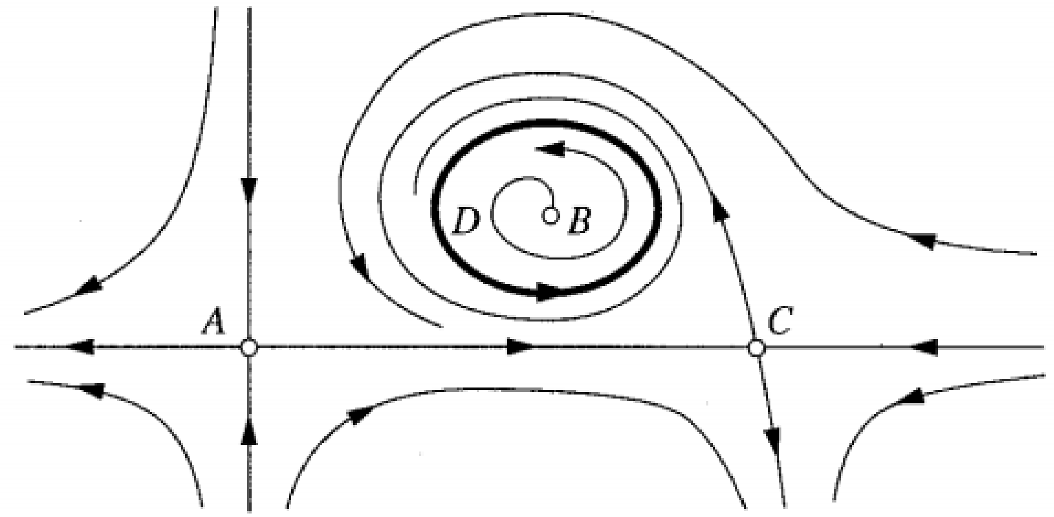


Figure 6.1.2

Poincaré-Bendixson Theorem

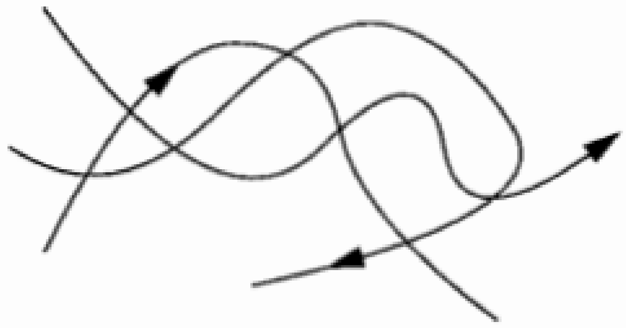


Figure 6.2.1

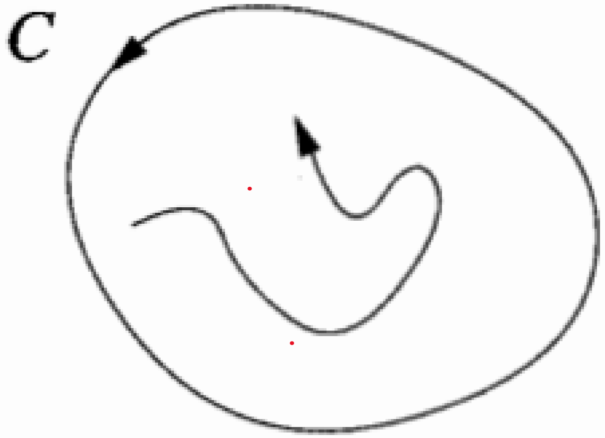


Figure 6.2.2

<u>Existence and Uniqueness Theorem</u>

- If *f* is continuously differentiable, then existence and uniqueness of solutions are guaranteed!

- The Poincaré-Bendixson Theorem states that, in a closed, bounded region where fixed points are not present, a trajectory MUST approach a closed orbit.
  - Ex: Hopf bifurcations (chemical oscillators), spirals, limit cycles, etc.

Future plans for research

- This research will continue to focus on 2-D dimensional flows
- In the future, the research focus will be on chaos theory and its applications
    - Applications may include weather prediction, biological inequalities, population growth, and more

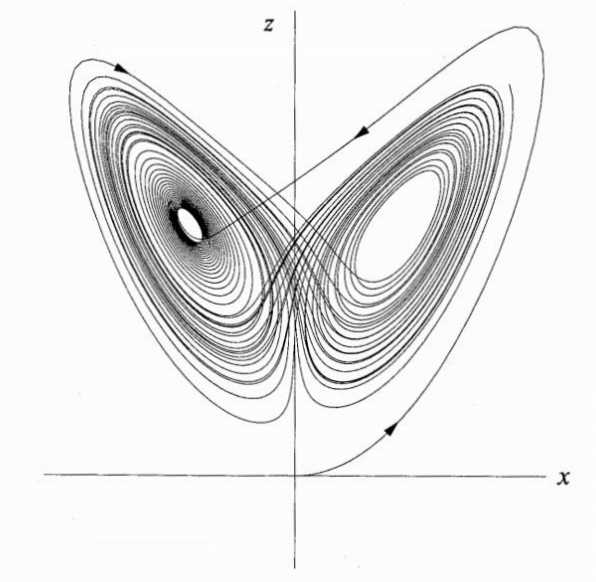
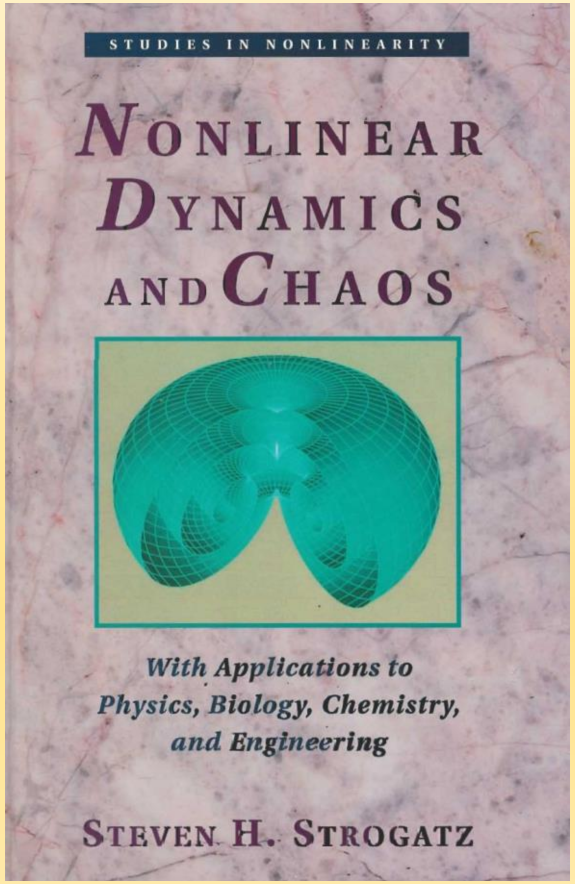


Figure 1.1.1

Reference

- Strogatz, S. H. (2015). *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, Second Edition (Studies in Nonlinearity)* (1st ed.). CRC Press.



Acknowledgements

- Thank you for listening to and/or reading my presentation

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